Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Differential Equations

Subject Code: 5SC01MTC2 **Branch: M.Sc. (Mathematics)**

Semester:1 **Date: 02/12/2015** **Time: 10:30To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a. Find the value of $\Gamma\left(\frac{7}{2}\right)$.

b. Write Legendre's equation.

c. Find radius of convergence of e^x.

d. Find the value of $J_0(0)$.

e. $\frac{d}{dx}[x^2J_2(x)] = x^2J_1(x)$. Determine whether the statement is true or false.

f. In Frobenius method, the equation formed from the coefficient of the lowest power of x is called indicial equation. Determine whether the statement is true or false.

g. Bessel's equation of order 4 is $x^2y'' + xy' + (x^2 - 4)y = 0$. Determine whether the statement is true or false.

Q-2 Attempt all questions

a. Solve $y''' - 6y'' + 11y - 6y = e^{2x}$ by variation of parameters. **(05)**

b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomial. **(05)**

c. Define: Ordinary and Regular singular point. Also determine whether $x = 0$ is an ordinary point or a regular singular point of the differential equation $2x^2y'' + 7x(x+1)y' - 3y = 0$. **(04)**

OR

Q-2 Attempt all questions

a. Find the solution in series of $y'' + xy' + x^2 y = 0$ about $x = 0$. **(05)**

b. If f be $n -$ times differentiable function on $[-1,1]$, then prove that **(05)**

$$\int_{-1}^{1} f(x)P_n(x)\ dx = \frac{(-1)^n}{2^n n!}\int_{-1}^{1} f^n(x)(x^2-1)^n\ dx.$$

c. Discuss the singularities of the equation $x^2y'' + xy' + (x^2 - n^2) = 0$ at $x = \infty$. **(04)**

Q-3 Attempt all questions

a. Verify that the origin is a regular singular point of **(07)**

$2x^2\frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0$ and find two independent Frobenius series solution of it.

b. Prove that if λ_m and λ_n are roots of the equation $J_p(a\lambda) = 0$, then **(07)**

$$\int_0^a xJ_p(\lambda_m x)J_p(\lambda_n x)\,dx = f(x) = \begin{cases} 0 \quad , & if m \neq n \\ \frac{a^2}{2}J_{p+1}^2(\lambda a), & if m = n \end{cases}$$

OR

Q-3 Attempt all questions

a. Prove that $\int_{-1}^{1} P_n(x)P_m(x)\ dx = \begin{cases} 0, & m \neq n \\ \frac{2}{2n+1}, & m = n \end{cases}$. **(07)**

b. State and prove Fourier-Bessel Series. Also prove that **(07)**

$$x^p = \sum_{m=1}^{\infty} \frac{2 \cdot a^{p-1}}{\lambda_m J_{p+1(\lambda_m a)}} J_p(\lambda_m x)\ dx$$

SECTION - II

Q-4 Answer the Following questions: **(07)**

a. Write Gauss's hyper geometric equation.

b. $J_n(x)$ and $J_{-n}(x)$ are orthogonal. Determine whether the statement is true or false.

c. Write clairaut's equation.

d. Define: Hypergeometric function.

e. Write auxiliary equation of Jacobi's method.

f. A Pfaffian differential equation in two variable is always integrable. Determine whether the statement is true or false.

g. $u = x^2 - y^2$ is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. Determine whether the statement is true or false.

Q-5 Attempt all questions

a. Using Picard's method of successive approximation, find a sequence of two functions which approach solution of the initial value problem, $y' = e^x + y^2, y(0) = 1$. **(05)**

b. Find a complete integral of $p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$. **(05)**

c. Prove that $(1 + x)^n = F(-n; 1; 1 - x)$. **(04)**

OR

Q-5 Attempt all questions

a. By Jacobi's method, solve the equation $u_x^2 + u_y^2 + u_z = 1$. **(05)**

b. Prove that let $u(x, y)$ and $v(x, y)$ be two functions of x and y such that $\frac{\partial v}{\partial y} \neq 0$. If, further $\frac{\partial(u,v)}{\partial(x,y)} = 0$, then there exists a relation $F(u, v) = 0$, between u and v not u and v not involving x and y explicitly. **(05)**

c. Eliminate the arbitrary function F form each of following equations and find the corresponding partial differential equation **(04)**

$i)\ z = F\left(\frac{xy}{z}\right)$, $\quad ii)\ F(xyz, x + y + z) = 0$.

Q-6 Attempt all questions

a. Find a complete integral of $(p^2 + q^2)y - qz = 0$ by Charpit's method. **(07)**

b. Verify that the Pfaffian differential equation **(07)**

$(y^2 + yz)\ dx + (xz + z^2)dy + (y^2 - xy)\ dz = 0$

is integrable and find the corresponding solution.

OR

Q-6 Attempt all Questions

a. Prove that a necessary and sufficient condition that the Pfaffian differential equation $\vec{X} \cdot d\vec{r} = P(x, y, z)\ dx + Q(x, y, z)\ dy + R(x, y, z)\ dz = 0$ be integrable is that $\vec{X} \cdot curl\ \vec{X} = 0$. **(07)**

b. Find the general solution of **(07)**

$i)\ (x^2 + y^2)p + 2xy\ q = (x + y)z$

$ii)\ (z^2 - 2yz - y^2)\ p + x(y + z)q = x(y - z)$